数理解析研究所講究録
999巻 1997年 92-115

The b-function of a prehomogeneous vector space $(SL(5)\times GL(4), \Lambda_2\otimes\Lambda_1)$.

埼玉大学理学部数学科
矢野環

概均質ベクトル空間 $(SL(5)\times GL(4), \Lambda_2\otimes\Lambda_1)$ の b-関数は、尾関育三によるデータをもとにして、矢野環・関口次郎により、軌道の局所的構造が決定され、さらにb関数の局所的双対性を用いて最終的に global な b関数が決定された。

以下には、以前に準備したこの最終段階の部分の原稿をおさめた。今ではb関数の双対性はさらに一般化されており、より見通しのよい議論も可能であるが、それはいずれ整理して発表する。又、以下の原稿はb関数の双対性を前提としている。その部分も別に発表する予定である。

末尾に holonomy diagram をつけた。これも1979年の尾関育三の論文(Proc. of Japan Acad. 37–40, +1 1979)のものを用いた。交わりは書き込まれるいくつか補充されている。(この図は私の手書きである)

Microlocal Structure of the Regular Prehomogeneous Vector Space Associated with SL(5) x GL(4). II.

By Tamaki Yano and Ikuzo Ozeki

The microlocal structure of the triplet (SL(5) x GL(4), $\Lambda_2 \otimes \Lambda_1$, V(10)⊗V(4)) is investigated in [O 1] and [O 2]. The triplet is a reduced irreducible prehomogeneous vector space (abbreviated as PV in the sequel), having 63 orbits. The b-function b(s) of this PV is of degree 40. The second author made a conjecture that

$$\begin{aligned} b(s) &= (s+1)^8\{(s+\tfrac{5}{6})(s+\tfrac{7}{6})(s+\tfrac{3}{4})(s+\tfrac{5}{4})\}^4\{(s+\tfrac{7}{10})(s+\tfrac{9}{10})(s+\tfrac{11}{10})(s+\tfrac{13}{10})\}^2 \\ &\quad \{(s+\tfrac{2}{3})(s+\tfrac{5}{3})\}^4 \\ &= \Pi_{k=1}^{3}(s+\tfrac{1}{2}+\tfrac{k}{4})^4\ \Pi_{k=1}^{4}(s+\tfrac{1}{2}+\tfrac{k}{5})^2\ \Pi_{k=1}^{5}(s+\tfrac{1}{2}+\tfrac{k}{6})^4 \end{aligned}$$

In this article, we will show that the b-functions of the orbits $S_{6,8}$, $S_{7,7}$ and $S_{12,10}$ are given by

$$\begin{aligned} b_{6,8}(s) &= (s+1)\ \Pi_{k=1}^{3}(s+\tfrac{1}{2}+\tfrac{k}{4})^2\ \Pi_{k=1}^{4}(s+\tfrac{1}{2}+\tfrac{k}{5})\ \ \Pi_{k=1}^{5}(s+\tfrac{1}{2}+\tfrac{k}{6}) , \\ b_{7,7}(s) &= \Pi_{k=1}^{3}(s+\tfrac{1}{2}+\tfrac{k}{4})^2\ \Pi_{k=1}^{4}(s+\tfrac{1}{2}+\tfrac{k}{5})\ \ \Pi_{k=1}^{5}(s+\tfrac{1}{2}+\tfrac{k}{6})^2 , \\ b_{12,10}(s) &= (s+1)^2\ \Pi_{k=1}^{3}(s+\tfrac{1}{2}+\tfrac{k}{4})^2\ \Pi_{k=1}^{2}(s+\tfrac{1}{2}+\tfrac{k}{3})^2 , \end{aligned}$$

and prove the Ozeki's conjecture.

Both $b_{6,8}(s)$, $b_{7,7}(s)$ and $b_{12,10}(s)$ coinside with micro-local b functions on corresponding holonomic varieties of those orbits.

In § 1, we prepare machinary from the microlocal calculus (see [SKKO]) and the locally prehomogeneous spaces (see [Y 3]).

Applying the latter, we get a duality

$$b_\Lambda(-s-2) = (-1)^{\deg b_\Lambda} b_\Lambda(s),$$

for each good holonomic variety Λ of our PV.

In § 2, we calculate $b_{6,8}(s)$ using the machinary in § 1.

In § 3, we calculate $b_{7,7}(s)$ and explain the backgroud of the conjecture.

In § 4, we collect the data of transverse localization, which will be needed in §§ 2-3.

The authors express our hearty gratitude to Prof. Tatsuo Kimura for his constant encouragement. We are grateful to Prof. Akihiko Gyoja for his enlightening discussions, which is the origin of this work.

§ 1. Preliminaries

Let (G,ρ,V) be an irreducible regular PV, and let f be its fundamental relative invariants. Let S be an orbit of (G,ρ,V). We denote by Λ_S (or T_S^*V) the closure of the conormal bundle of S, and call it the holonomic variety.

We refer the reader [SKKO] for detailed discussion of micro-local calculus. We quote the following Theorem.

Theorem 1.1 ([Theorem 7.5 in SKKO]). Let Λ_0 and Λ_1 be good holonomic varieties whose intersection is of codimension one with the intersection exponent $(m:n)$. Assume that $\mathcal{M} = \mathcal{E}f^{\alpha}$ is a simple holonomic system with support $\Lambda_0 \cup \Lambda_1$. Assume that $m_0 > m_1$ where $\mathrm{ord}_{\Lambda_i} f^{\alpha} = -m_i\alpha - \mu_i/2$. Then we have

$$b_{\Lambda_0}(s)/b_{\Lambda_1}(s) = \Pi_{k=0}^{n} \left[\frac{1}{n+1}(\mathrm{ord}_{\Lambda_1} f^{\alpha} - \mathrm{ord}_{\Lambda_0} f^{\alpha})\Big|_{\alpha=s} + \frac{m+2k}{2(m+n)} \right]^{\frac{m_0-m_1}{n+1}}$$

where $[a]^b = a(a+1)\dots(a+b-1)$.

Let $(\mathcal{G},U)$ be a locally prehomogeneous space that is weighted homogeneous and logarithmically free (abbrev. WH LGF LPH). We refer the reader [Y 3] for the micro-local calculus of LPH.

Set $\mathcal{N}(s) = \mathcal{D}[s] / \mathcal{J}(s)$, $\mathcal{J}(s) = \{P(s) \in \mathcal{D}[s] \; ; \; Pf^s = 0\}$,

$\mathcal{N}_{\alpha} = \mathcal{D} / \mathcal{J}(\alpha)$, $\mathcal{J}(\alpha) = \{Q \in \mathcal{D} \; ; \; Q = P(\alpha),\ P(s) \in \mathcal{J}(s)\}$,

$\mathcal{M}(s) = \mathcal{N}(s) / \mathcal{N}(s+1)$.

We define

$b_{(S)}$= the minimal polynomial of s in $\mathcal{H}om_{\mathcal{D}}(\mathcal{M}(s),\mathcal{B}_{S|U})_p$ $p \in S$,

where $\mathcal{B}_{S|U}$ is the sheaf of delta functions supported on S.

Let S be an orbit at $0 \in U$. Set

$\mathscr{C}(S) = \{ S' \; ; \; S'$ is a local orbit with $S \subset \overline{S'} \}$,

$$b_{<\Lambda_S>} = \underset{\substack{S' \in \mathscr{C}(S) \\ S':\text{good}\ ,\ S' \neq S}}{\text{l.c.m.}} (b_{\Lambda_{S'}}) ,$$

$$b_{<S>} = \underset{\substack{S' \in \mathscr{C}(S) \\ S' \neq S}}{\text{l.c.m.}} (b_{S'}) ,$$

$$b^{g}_{(S)} = \prod_{k=1}^{\text{codim}\, S} \underset{\substack{S' \in \mathscr{C}(S),\text{codim}\, S'=k \\ S':\text{good}\ ,\ S' \neq S}}{\text{l.c.m.}} (b_{(S')}) ,$$

$$b^{ng}_{(S)} = \prod_{k=1}^{\text{codim}\, S} \underset{\substack{S' \in \mathscr{C}(S),\text{codim}\, S'=k \\ S':\text{not good}\ ,\ S' \neq S}}{\text{l.c.m.}} (b_{(S')}) ,$$

The following theorem is proved in [Y 3].

Theorem 1.2. Let $(\mathscr{G},U)$ be a WH LGF LPH with finite orbits. Then, the following properties hold.

(1) Let S be a good orbit. Then $b_{\Lambda_S} \mid b_S$.

(2) For all good holonomic variety Λ , we have

$$(1.1) \quad b_{\Lambda}(-s-2) = (-1)^{\deg b_{\Lambda}} b_{\Lambda}(s).$$

(3) Let S be an orbit. Then,

(1.2) $b_S \quad | \text{ l.c.m.} \left(b_{<\Lambda_S>} , b_{(S)} b^{ng}_{(S)} \right)$.

If S is good, we have also

(1.3) $b_S \quad | \text{ l.c.m.} \left(b_{\Lambda_S} , b_{<\Lambda_S>} , b^{ng}_{(S)} \right)$.

If $\mathscr{C}(S)$ consists of good orbits, we have

(1.4) $b_S = \text{l.c.m.} \left(b_{\Lambda_S} , b_{<\Lambda_S>} \right)$.

(4) $b_{<S>} \mid b_S \quad , \quad b_{(S)} \mid b_S$.

Theorem 1.3. [Prop. Y3] Let S be a good orbit of LGF LPH. Then, if $(s + 1) \mid b_{(S)}$, we have

$$m_S - \operatorname{codim} S \in 2\mathbb{Z} .$$

Theorem 1.4. Let S be an orbit of PV $(SL(5)xGL(4), \Lambda_2 \otimes \Lambda_1, V)$.

(1) [Thm 3.4 in O 2] If Λ_S is G-prehomogeneous, then S is good.

(2)[O 1, Prop in Y 3] $\quad \operatorname{ord}_{\Lambda_S} f^{\alpha} = - m_S \left(\alpha + \frac{1}{2} \right) , \; m_S \geq 0$.

Our main concern is the orbits appearing in the following Figure 1. Each orbit $S_{i\ j}^{\ k}$ (if k=0, k is omitted) corresponds to a vertex encircling $i\, {}^{k}_{j}$. See [O 1] for details. Orbits $S_{5,21}$ and $S_{6,14,2}$ are not good. All the others are good orbits.

§ 2. B-functions .

In this section, we will determine micro-local b-functions and b-functions of orbits appearing in the following holonomy diagram (Figure 1, extracted from [O 1]) except $S^1_{5,21}$ and $S^2_{6,14}$ which are not good.

Theorem 2.1. Let S be an good orbit appearing in Figure 1. The micro-local b-function is given by the following Table 1. Here, if $b_\Lambda = \Pi_{i=1}^{m}(s + \alpha_i)$, we exhibit $\{ \alpha_i \}_i$. The underlined factor $\underline{\alpha}$ denotes the fact that $b_{(S)} = \prod_{\alpha:\text{underlined}} (s + \alpha)$.

m	orbit	α_i
0	0 40	ϕ
1	1 30	$\underline{1}$
2	2 24	$\underline{1}$ 1
3	2 21	1 $\underline{5/6}$ $\underline{7/6}$
6	3 15	1 1 5/6 7/6 $\underline{3/4}$ $\underline{5/4}$
4	3 18	1 1 5/6 7/6
4	4 20	$\underline{1}$ 1 5/6 7/6
5	5 16	$\underline{1}$ 1 1 5/6 7/6
8	4 14	$\underline{1}$ 1 5/6 7/6 $\underline{3/4}$ $\underline{5/4}$ 3/4 5/4
9	5 12	$\underline{1}$ 1 1 5/6 7/6 3/4 5/4 3/4 5/4
10	4 11	1 1 5/6 7/6 3/4 5/4 $\underline{7/10}$ $\underline{9/10}$ $\underline{11/10}$ $\underline{13/10}$
10	6 14	1 1 5/6 7/6 5/6 7/6 3/4 5/4 3/4 5/4
15	5 9	$\underline{1}$ 1 1 5/6 7/6 3/4 5/4 3/4 5/4 7/10 9/10 11/10 13/10 $\underline{2/3}$ $\underline{4/3}$
16	6 8	$\underline{1}$ 1 1 1 5/6 7/6 3/4 5/4 3/4 5/4 7/10 9/10 11/10 13/10

							2/3	4/3						
11	8	11		1	1	1	5/6	7/6	5/6	7/6	3/4	5/4	3/4	5/4
12	12	10	$\underline{1}$	1	1	1	5/6	7/6	5/6	7/6	3/4	5/4	3/4	5/4
20	7	7	1	1	1	1	5/6	7/6	5/6	7/6	3/4	5/4	3/4	5/4
							7/10	9/10	11/10	13/10	2/3	4/3	2/3	4/3

Table 1.

The proof is given simultaneously in the proof of the following:

Theorem 2.2. Let S be a good orbit appearing in Fig.1.

(1) If $S \neq S_{6,14}$, we have

$b_S = b_{\Lambda_S}$, where b_{Λ_S} is given by Table 1.

(2) $b_{6,14} = (s+1)\, b_{\Lambda_{6,14}}$.

We prove theorems for $S \neq S_{7,7}$ in this section. The proof for $S_{7,7}$ will be given in § 3.

Proof. We proceed in 6 steps.

Step 1. $S = S_{0,40}, S_{1,30}, S_{2,24}, S_{2,21}, S_{3,18}, S_{4,20}, S_{5,16}$.

b_Λ is known by Thm 1.1.

Owing to Thm 1.2 (1.4), we have $b_\Lambda = b_S$.

Step 2. $S = S_{3,15}, S_{4,11}$.

The transverse localization along S is the discriminant $d_4(1,0,x_2,x_3,x_4)$ (resp. $d_5(1,0,x_2,x_3,x_4,x_5)$). We determine

$b_{(S)}$ by the method explained in [YS].

$b_{(3,15)} = (s+\frac{3}{4})(s+\frac{5}{4})$

$b_{(4,11)} = \Pi_{k=0}^{3}(s+\frac{7+2k}{10})$.

By Thm 1.2 (1.4) and (4), we get b_S. Since b_S satisfies deg b_S =

m_S , we have $b_S = b_\Lambda$.

Step 3. $S = S_{4,14}, S_{5,12}$.

First, set $S = S_{4,14}$. Using the transverse localization in § 4, we know that

$$b_{(4,14)} = (s+1)(s+\tfrac{3}{4})(s+\tfrac{5}{4}).$$

Now set $b' = (s+1)^2(s+\tfrac{5}{6})(s+\tfrac{7}{6})(s+\tfrac{3}{4})^2(s+\tfrac{5}{4})^2$.

Owing to Thm 1.2 (1) and (1.2), we get

$$b_\Lambda \mid b_{4,14} \mid (s+1)b' \quad .$$

Therefore, by Thm 1.2 (2) and the fact that deg b_Λ=8, we get b_Λ= b'. By Thm 1.2 (1.4), we have $b_{4,14} = b_\Lambda$.

Next, set $S = S_{5,12}$. Since $b_{\Lambda_{5,12}} / b_{\Lambda_{4,14}} = (s+1)$, we get $b_{\Lambda_{5,12}}$. By Thm 1.2 (1.4) , $b_{5,12} = b_{\Lambda_{5,12}}$.

Step 4. $S = S_{5,9}$.

We can show directly that the possible α such that $(s+\alpha)|b_{(S)}$ and $\alpha \le 1$ is , counting multiplicity, $\alpha = \frac{2}{3}$, 1 .

Set $b' = (s+1)^2(s+5/6)(s+7/6)(s+3/4)^2(s+5/4)^2(s+7/10)(s+9/10)$

$(s+11/10)(s+13/10)(s+2/3)(s+4/3)$.

Then, using Thm 1.2 (1.2) and (4), we see that $b' \mid b_S \mid (s+1)b'$. Since deg b' = 14, deg b_Λ = 15 and $b_\Lambda \mid b_S$, we have $b_\Lambda = b_S = (s+1)b'$.

Thus we get b_Λ and by Thm 1.2 (1.4) $b_S = b_\Lambda$.

Step 5. $S = S_{6,8}$.

By Thm 1.1, $b_{\Lambda_{6,8}} = (s+1)b_{\Lambda_{5,9}}$. By Thm 1.2 (1.4), we get $b_S = b_\Lambda$.

Step 6. $S = S_{6,14}$, $S_{8,11}$, $S_{12,10}$.

Since $b_{\Lambda_{8,11}}/b_{\Lambda_{5,12}} = (s+\tfrac{5}{6})(s+\tfrac{7}{6})$, $b_{\Lambda_{8,11}}/b_{\Lambda_{6,14}} = (s+1)$, and $b_{\Lambda_{12,10}} / b_{\Lambda_{8,11}} = (s+1)$, we get $b_{\Lambda_{8,11}}$, $b_{\Lambda_{6,14}}$ and $b_{\Lambda_{12,10}}$.

Since the transverse localization along $S_{12,10}$ is isomorphic to a regular irreducible PV $(SL(3)xGL(2), 2\Lambda_1 \otimes \Lambda_1, V)$ (cf.[O 2]), we have $b_{12,10} = b_{\Lambda_{12,10}}$.

We do not know whether $\Lambda_{5,21} \subset W$ or not. But, we can calculate that $b_{(5,21)} | (s+5/6)(s+7/6)$.

Applying Thm 1.2 (1.3) and (4) on $S_{6,14}$, we get $b_{S_{6,14}} = (s+1) b_{\Lambda_{6,14}}$. Especially, we also proved that $\Lambda_{5,21} \subset W$ and $b_{(5,21)} = (s+5/6)(s+7/6)$.

Then, by Thm 1.2 (1) and (1.3) proves $b_{8,11} = b_{\Lambda_{8,11}}$.

Hence the result.

Q.E.D.

Remark 2.3. From the orbits $S_{1,30}$, $S_{2,21}$, $S_{3,15}$, $S_{4,11}$, $S_{5,9}$, we get factors $s + \frac{1}{2} + \frac{1}{k}$, $k = 2,3,4,5,6$ respectively. The first four orbits give discriminants $d_k(1,0,x_2,..,x_k)$ whereas the last is not.

Remark 2.4. The b-function of $(SL(3)xGL(2), 2\Lambda_1 \otimes \Lambda_1, V)$ is anounced in [KM]. However, they used the 3-Lagrangeans formula, whose proof has not yet published. The above proof does not appeal to the 3-Lagrangeans formula, and hence gives another proof of the b-function of that PV.

§ 3. B-functions of $S_{7,7}$

The orbit $S_{7,7}$ intersects with $S_{6,8}$, $S_{5,9}$, $S_{4,11}$ and $S_{5,12}$ at one place on $\Lambda_{7,7}$ and with $S_{6,14,2}$ at another place.

The former intersection is so complicated and we cannot apply standard method of micro-local calculus. As for the latter, $S_{6,14,2}$ is not a simple holonomic variety.

Assume that $\Lambda_{6,14,2} \subset W$. Using the structure of the isotropy subalgebra at a point of $S_{6,14,2}$, we can see that

(3.1) $\quad b_{(6,14,2)} \mid b'(s)$

where $\quad b'(s)=(s+1)(s+2/3)(s+4/3)(s+5/6)(s+7/6)$.

We can also see, using the data in 4.5 and applying Thm 1.3,

(3.2) $\quad b_{(7,7)} = 1$.

Consider $\quad \mathcal{M}' = b_{5,9}(s)\mathcal{M}(s)$. Then, using the result in §2, we see that $\quad$ codim $\mathrm{Supp}(\mathcal{M}') \geq 6$ near a generic point of $S_{7,7}$. Since $b_{(6,8)}=(s+1)$, codim $\mathrm{Supp}(\ b'(s)\mathcal{M}') \geq 7$. Then, by (3.2), $b_{7,7} \mid b'b_{5,9}$. Since $b_{\Lambda_{7,7}} \mid b_{7,7}$ and deg $b_{\Lambda_{7,7}}=20$, we get $b_{\Lambda_{7,7}}=b_{7,7}=b'b_{5,9}$. Note that we also proved $\Lambda_{6,14,2} \subset W$.

Hence, we proved Thm 2.1 and Thm 2.2 for $S_{7,7}$.

Corollary 3.1. (1) $\Lambda_{5,21}$, $\Lambda_{6,14,2} \subset W$.

(2) Suppose $b_{5,21}(-s-2)= \pm b_{5,21}(s)$ and $b_{6,14,2}(-s-2)=\pm b_{6,14,2}(s)$.

Then,

$$b_{5,21} = (s+1)^2(s+3/4)(s+5/4)\{(s+5/6)(s+7/6)\}^2 ,$$

$$b_{6,14,2} =(s+1)^{3+a}(s+2/3)(s+4/3)\{(s+3/4)(s+5/4)(s+5/6)(s+7/6)\}^2,$$

where $a=0$ or 1.

Proof. (1) has already proved. Using Thm 1.2 (1.3),

$$b_{5,21} = (s+1)^2(s+3/4)(s+5/4)\{(s+5/6)(s+7/6)\}^{1+b} ,$$

where $b = 0$ or 1. Suppose $b=0$. Then, if we set

$$b' = (s+1)^3\{(s+3/4)(s+5/4)\}^2(s+5/6)(s+7/6) ,$$

we see that codim Supp$(b'\mathcal{M}(s)) \geq 6$ at a generic point of $S_{6,14}$. Since $b_{(6,14)}=1$, we have $b_{6,14}|b'$, which is a contradiction because $\deg b'=9 < 11=\deg b_{6,14}$. The proof for $b_{6,14,2}$ is similar.

Q.E.D.

Quite generally, when (G,ρ,V) is a reduced irreducible regular PV other than ours, for any good Lagrangean Λ and its dual $\Lambda^{\vee}$, we have

$$(3.3) \qquad b_{\Lambda^{\vee}}(-s-\frac{\dim V}{\deg f}-1) = \pm\ b(s)/b_{\Lambda}(s) .$$

This global duality is proved in [Y 3] using Kashiwara's lemma. Then, combining with Thm 1.2 (2), we have

$$(3.4) \qquad b(s) = \{b_{\Lambda_{7,7}}(s)\}^2 .$$

Therefore, the following Ozeki's conjecture is proved.

$$(3.5) \qquad b(s) = (s+1)^8\{(s+3/4)(s+5/4)\}^4\{(s+5/6)(s+7/6)\}^4 \\ \{\Pi_{k=0}^{3}(s+\frac{7+2k}{10})\}^2\{(s+2/3)(s+4/3)\}^4 .$$

There is another approach to prove (3.5). We quote the recent results of Professors K.Kawanaka and A.Gyoja.

Proposition 3.2. (S.Kawanaka)

(1) $(s+1)^8$ divides $b(s)$ but $(s+1)^9$ does not.

(2) $(s+\frac{1}{2})$ or $(s+\frac{3}{2})$ dose not divide $b(s)$.

Proposition 3.3. (A.Gyoja)

Set $b^{\exp}(t) = \Pi_{i=1}^{40}(t-\exp(2\pi\sqrt{-1}\,\alpha_i))$, for $b(s)=\Pi_{i=1}^{40}(s+\alpha_i)$.

(1) $b^{\exp}(t)$ is a product of cyclotomic polynomials.

(2) If we denote by f_n the n-th cyclotomic polynomial,

$$b^{\exp}(t) \mid f_1^8 f_2^8 f_3^4 f_4^4 f_5^2 f_6^4 f_7 f_8^2 f_9 f_{10}^2 f_{12}^2 f_{14} f_{15} f_{18} f_{20} f_{24} f_{30}.$$

Suppose that we get an estimate of the following form.

$$(3.6)\quad b(s) \mid (s+1)^{a_1}\{(s+3/4)(s+5/4)\}^{a_4}\{(s+5/6)(s+7/6)\}^{a_6}$$
$$\{\Pi_{k=0}^{3}(s+\tfrac{7+2k}{10})\}^{a_{10}}\{(s+2/3)(s+4/3)\}^{a_3},$$

where $a_1,\dots,a_{10}$ are non-negative integers.

Then, Prop.3.3 (1)(2) and (3.6) prove (3.5) , because (3.6) leads

$$(3.7)\quad b^{\exp}(t) \mid f_1^{a_1} f_3^{a_3} f_4^{a_4} f_6^{a_6} f_{10}^{a_{10}},$$

and counting degree's , we must have $a_1=8$, $a_3=4$, $a_4=4$, $a_6=4$, $a_{10}=2$.

Using a result in [Y 3] and Prop.3.3 (1) we can show that the factor of $b^{\exp}(t)$ derived from good orbits is at most

$$f_1^{13} f_3^2 f_4^2 f_6^2 f_{10}.$$

Therefore, we must study non good orbits.

The related topics will be discussed in the forthcoming article.

Note: We get an estimate (3.6) with $a_1=20$, $a_3=4$, $a_4=5$, $a_6=6$, $a_{10}=2$.

§ 4. Transverse localizations

In this section, we list up transverse localizations needed in § 2. For that of $S_{4,11}$ the reader should refer [Y 1] or [O 2].

In the following paragraphs, v_0 denotes a point of the orbit, $p(x)$ a transverse direction and $L(x)$ denotes the coefficients of a basis of $\mathscr{H}$ of the transverse localization $(\mathscr{H},H)$ along S at v_0

$\sigma(f^{\alpha})$: the principal symbol of f^{α} on $T^{*}_{\{v_0\}}H$,

$v_{12,10}$: a point of $S_{12,10}$, $v_{12,10}=236 - 137 + 128 + 459$.

4.1. $S_{6,8}$

$v_0 = 256 - 346 + 157 - 148 + 238 - 129$.

basis of $T^{*}_{v_0}V$: 247, 359, 459, 259 + 349 - 458,

457 - 249 , 239 + 149 - $\frac{1}{2}$(348+258-456+3<357>).

$$p(x) = -\frac{2}{3}x_2\langle 239\rangle+x_3\langle 249\rangle-\frac{4}{3}x_4\langle 259\rangle+\frac{4}{3}x_5\langle 359\rangle+8x_6\langle 459\rangle-x_0\langle 247\rangle$$

$$L(x)=\begin{pmatrix}
2x_2 & 0 & 3x_3 & 4x_4 & 5x_5 & 6x_6 \\
3x_3 & x_3 & x_0(4x_4-\frac{4}{3}x_2^2) & \frac{40}{3}x_0x_5-\frac{2}{3}x_2x_3 & 16x_6-\frac{16}{9}x_2x_4 & -\frac{8}{9}x_0x_2x_5+\frac{1}{9}x_3x_4 \\
4x_4 & 0 & 5x_0x_5-x_2x_3 & 36x_6-\frac{4}{3}x_2x_4 & -2x_2x_5 & \frac{4}{9}x_4^2-\frac{4}{3}x_3x_5+\frac{8}{3}x_2x_6 \\
5x_5 & -x_5 & 6x_6-\frac{2}{3}x_2x_4 & -2x_2x_5 & 0 & -\frac{1}{9}x_4x_5 \\
6x_6 & 0 & -\frac{1}{3}x_2x_5 & \frac{4}{9}x_4^2-\frac{4}{3}x_3x_5+\frac{8}{3}x_2x_6 & -\frac{1}{9}x_4x_5 & \frac{2}{27}(-5x_0x_5^2+12x_4x_6+4x_2^2x_6+\frac{1}{3}x_2x_4^2-x_2x_3x_5) \\
0 & 2x_0 & 0 & 0 & -x_3 & 0
\end{pmatrix}$$

$${}^{t}(X_{0\cdot},\ X_{\cdot 0},\ X_{11},\ X_{20},\ X_{3-1},\ X_{40}) = {}^{t}L(x)\,{}^{t}(\partial_2,\partial_3,\partial_4,\partial_5,\partial_6,\partial_0),$$

	$X_{0\cdot}$	$X_{\cdot 0}$	X_{11}	X_{20}	X_{3-1}	X_{40}
$X(\log f)$	30	2	0	$-\frac{4}{3}x_2$	0	$2x_4+\frac{8}{27}x_2^2$
$\mathrm{div}^{(x)}X$	20	2	0	$-\frac{4}{3}x_2$	0	$\frac{16}{9}x_4+\frac{8}{27}x_2^2$
$\mathrm{Tr}^B_{\mathfrak{g}}\ \mathrm{ad}\ X$	10	0	0	0	0	$\frac{2}{9}x_4$

$$
\begin{aligned}
&[X_{0\cdot},X_{ij}] = iX_{ij}\ ,\ [X_{\cdot 0},X_{ij}] = jX_{ij}\ ,\\
&[X_{11},X_{20}] = -4x_0X_{3-1} - \tfrac{2}{3}x_2X_{11} - x_3X_{\cdot 0},\\
&[X_{11},X_{3-1}] = -3X_{40} + \tfrac{1}{3}x_2X_{20} - \tfrac{1}{2}x_3X_{11} + \tfrac{1}{3}x_4X_{0\cdot} + 4(x_4 - \tfrac{1}{3}x_2^2)X_{\cdot 0},\\
&[X_{11},X_{40}] = \tfrac{1}{6}x_3X_{20} - \tfrac{1}{3}x_4X_{11} + \tfrac{1}{3}x_5x_0X_{0\cdot},\\
&[X_{20},X_{3-1}] = -\tfrac{2}{3}x_2X_{3-1} + \tfrac{2}{3}x_5X_{0\cdot} + \tfrac{40}{3}x_5X_{\cdot 0},\\
&[X_{20},X_{40}] = \tfrac{2}{9}x_4X_{20} - \tfrac{8}{9}x_5X_{11} + \tfrac{8}{3}x_6X_{0\cdot},\\
&[X_{3-1},X_{40}] = -\tfrac{1}{9}x_4X_{3-1} + \tfrac{1}{9}x_5X_{20} - \tfrac{8}{9}x_2x_5X_{\cdot 0}\ .
\end{aligned}
$$

Set $X_3' = x_0X_{3-1} + \frac{1}{2}x_3X_{\cdot 0}$. Then we have

$$
\begin{aligned}
&[X_{0\cdot},X_3'] = 3X_3'\ ,\\
&[X_{11},X_{20}] = -4x_0X_3' - \tfrac{2}{3}x_2X_{11}\ ,\\
&[X_{11},X_3'] = x_0\{-3X_{40} + \tfrac{1}{3}x_2X_{20}\} - \tfrac{1}{2}x_3X_{11} + \tfrac{1}{3}x_0x_4X_{0\cdot}\ ,\\
&[X_{20},X_3'] = -\tfrac{2}{3}x_2X_3' + \tfrac{2}{3}x_0x_5X_{0\cdot}\ ,\\
&[X_3',X_{40}] = -\tfrac{1}{9}x_4X_3' + \tfrac{1}{9}x_0x_5X_{20}\ .
\end{aligned}
$$

$$\sigma(f^{\alpha}) = \xi_0^{-\alpha-1}\ \xi_2^{-15\alpha-10}\ \sqrt{d\xi}\ /\ \sqrt{dx}\ .$$

$D_2^5\delta$ is an eigenfunction belonging to an eigenvalue -1.

$$b_{(6,8)} = s + 1\ .$$

4.2. $S_{5,9}$

$v_0 = 256 - 346 + 157 - 247 - 148 + 238 - 129$.

basis of $T^*_{v_0}V$: 359, 459, 259 + 349 - 458,

159 + 249 - 457 - 2<358>, 239 + 149 - $\frac{1}{2}$(348+258-456+3<357>).

$p(x) = -\frac{2}{3}x_2\langle 239\rangle + x_3\langle 249\rangle - \frac{4}{3}x_4\langle 259\rangle + \frac{4}{3}x_5\langle 359\rangle + 8x_6\langle 459\rangle$

The localization is given by setting $x_0=1$ in 4.1.

$$L(x)=\begin{pmatrix} 2x_2 & 3x_3 & 4x_4 & 5x_5 & 6x_6 \\ 3x_3 & 4x_4-\frac{4}{3}x_2^2 & \frac{40}{3}x_5-\frac{2}{3}x_2x_3 & 16x_6+\frac{1}{2}x_3^2-\frac{16}{9}x_2x_4 & -\frac{8}{9}x_2x_5+\frac{1}{9}x_3x_4 \\ 4x_4 & 5x_5-x_2x_3 & 36x_6-\frac{4}{3}x_2x_4 & -2x_2x_5 & \frac{4}{9}x_4^2-\frac{4}{3}x_3x_5+\frac{8}{3}x_2x_6 \\ 5x_5 & 6x_6-\frac{2}{3}x_2x & -2x_2x_5 & -\frac{1}{2}x_3x_5 & -\frac{1}{9}x_4x_5 \\ 6x_6 & -\frac{1}{3}x_2x_5 & \frac{4}{9}x_4^2-\frac{4}{3}x_3x_5+\frac{8}{3}x_2x_6 & -\frac{1}{9}x_4x_5 & \frac{2}{27}(-5x_5^2+12x_4x_6+4x_2^2x_6 \\ & & & & +\frac{1}{3}x_2x_4^2-x_2x_3x_5) \end{pmatrix}$$

$${}^t(X_0, X_1, X_2, X_3, X_4) = {}^tL(x)\,{}^t(\partial_2, \partial_3, \partial_4, \partial_5, \partial_6),$$

	X_0	X_1	X_2	X_3	X_4
$X(\log f)$	30	0	$-\frac{4}{3}x_2$	x_3	$2x_4+\frac{8}{27}x_2^2$
$\mathrm{div}^{(x)}X$	20	0	$-\frac{4}{3}x_2$	$\frac{1}{2}x_3$	$\frac{16}{9}x_4+\frac{8}{27}x_2^2$
$\mathrm{Tr}^B_{\mathfrak{g}}\,\mathrm{ad}\,X$	10	0	0	$\frac{1}{2}x_3$	$\frac{2}{9}x_4$

where $f = \det L(x)$.

$$[X_0,X_k] = kX_k \quad , \ k=0,1,\dots,4,$$
$$[X_1,X_2] = -4X_3 - \frac{2}{3}x_2X_1,$$
$$[X_1,X_3] = -3X_4 + \frac{1}{3}x_2X_2 - \frac{1}{2}x_3X_1 + \frac{1}{3}x_4X_0,$$
$$[X_1,X_4] = \frac{1}{6}x_3X_2 - \frac{1}{3}x_4X_1 + \frac{1}{3}x_5X_0,$$
$$[X_2,X_3] = -\frac{2}{3}x_2X_3 + \frac{2}{3}x_5X_0,$$
$$[X_2,X_4] = \frac{2}{9}x_4X_2 - \frac{8}{9}x_5X_1 + \frac{8}{3}x_6X_0,$$
$$[X_3,X_4] = -\frac{1}{9}x_4X_3 + \frac{1}{9}x_5X_2 .$$

$$\sigma(f^{\alpha}) = \xi_2^{-15\alpha-10}\ \sqrt{d\xi}\ /\ \sqrt{dx} \qquad .$$

$$b_{(3,15)} = (s+1)(s+\tfrac{2}{3})(s+\tfrac{4}{3}).$$

4.3. $S_{4,14}$

$v_0 = v_{12,10} + 146 + 257$.

basis of $T^*_{v_0}V$: 347 + 248, 348, 358, 356 + 158

$p(x) = x_0(\langle 347\rangle+\langle 248\rangle)+ x_2\langle 348\rangle+ x_4\langle 358\rangle+x_3(\langle 356\rangle+\langle 158\rangle)$.

$$L(x) = \begin{pmatrix} 0 & 3x_0 & 0 & x_2 \\ 2x_2 & 4x_2 & 6x_0^2x_3 & 4x_0x_4 \\ 4x_4 & 2x_4 & 4x_2x_3 & 6x_0x_3^2 \\ 3x_3 & 0 & x_4 & 0 \end{pmatrix} ,$$

$${}^t(X_{0\cdot}\ ,\ X_{\cdot 0}\ ,\ X_{12}\ ,\ X_{21}) = {}^tL(x)\,{}^t(D_0\ ,\ D_2\ ,\ D_4\ ,\ D_3\ ) ,$$

	$X_{0\cdot}$	$X_{\cdot 0}$	X_{12}	X_{21}
$X(\log f)$	12	12	0	0
$\mathrm{div}^{(x)}X$	9	9	0	0
$\mathrm{Tr}^B_{\mathfrak{g}}$ ad X	3	3	0	0

where $f = \det L(x)$.

$$[X_{0\cdot},X_{12}] = X_{12}\ ,\ [X_{0\cdot},X_{21}] = 2X_{21},$$
$$[X_{\cdot 0},X_{21}] = X_{21}\ ,\ [X_{\cdot 0},X_{12}] = 2X_{12},$$
$$[X_{12},X_{21}] = 2x_0x_3\ (\ X_{0\cdot} - X_{\cdot 0}).$$

The f(x) is the discriminant $d_3(x_0^2\ ,\ x_2\ ,\ x_4\ ,\ x_3^2\ )$ of the binary cubic form $x_0^2\ u^3 + x_2u^2v + x_4uv^2 + x_3^2\ v^3$.

$$f(x) = 12(\ x_2^2x_4^2 -4x_0^2x_4^3 -4x_2^3x_3^2 +18x_0^2x_2x_3^2x_4 - 27x_0^4x_3^4\ ) .$$

$$\sigma(f^{\alpha}) = (\xi_0\xi_3)^{-4\alpha-3} \sqrt{d\xi} / \sqrt{dx} .$$

We can see directly that

$$b_{(4,14)} = (s+1)(s+\tfrac{3}{4})(s+\tfrac{5}{4}).$$

Here, eigenvectors δ, $D_0D_3\delta$, $(D_0^2D_3^2 - 24D_2D_4)\delta$ belong to eigenvalues $-3/4$, -1, $-5/4$, respectively.

4.4. $S_{3,15}$

Under the situation in $S_{4,14}$, set $x_0 = 1$. Then, we get a localization along $S_{3,15}$.

$$L(x) = \begin{pmatrix} 2x_2 & 3x_3 & 4x_4 \\ 3x_3 & \frac{1}{2}x_4 & 2x_2x_3 \\ 4x_4 & 2x_2x_3 & 6x_3^2 + 2x_2x_4 \end{pmatrix} .$$

$${}^t(X_0, X_1, X_2) = {}^tL(x)\ {}^t(D_2, D_3, D_4).$$

	X_0	X_1	X_2
$X(\log f)$	12	0	$4x_2$
$\mathrm{div}^{(x)}X$	9	0	$4x_2$
$\mathrm{Tr}_{\mathfrak{g}}^{B}$ ad X	3	0	0

where $f = \det L(x) = 2(-4x_4^3 + x_2^2x_4^2 + 8x_2x_3^2x_4 - 27x_3^4 - 4x_2^3x_3^2)$.

$$[X_0, X_k] = kX_k , \quad [X_1, X_2] = x_3X_0 .$$

$$\sigma(f^{\alpha}) = \xi_2^{-6\alpha - 9/2} \sqrt{d\xi} / \sqrt{dx} .$$

$f(x) = d_3(1, x_2, x_4, x_3^2)$ is the discriminant of type D_3, hence isomorphic to that of type A_3 . This polynomial was treated in [YS I, §6], and we proved the following.

$$b_{(3,15)} = (s+\tfrac{3}{4})(s+\tfrac{5}{4}),$$

$$b_{3,15} = (s+1)(s+\tfrac{5}{6})(s+\tfrac{7}{6})b_{(3,15)} .$$

4.5. $S_{7,7}$

The transverse localization is determined by J.Sekiguchi and the second author.

$$v_0 = 356 + 137 + 128 + 458 + 149 + 239$$

Basis of $T^*_{v_0}V$: 246, 247, 257, 157-259, 256-127+457, 236-248+347-146, $2\langle 456\rangle - 2\langle 126\rangle$-147+237-249.

$$p(x)=x_{20}\langle 157\rangle+x_{11}\langle 237\rangle+x_{22}\langle 247\rangle+x_{31}\langle 257\rangle+x_{01}\langle 347\rangle+x_{21}\langle 457\rangle+x_{12}\langle 246\rangle.$$

$$L(x) = \left(\begin{array}{ccccc} 0 & 2x_{20} & 0 & x_{21}+x_{20}x_{01} & x_{31}-x_{20}x_{11} \\ x_{01} & 0 & x_{11}/2 & 0 & -3x_{12} \\ x_{11} & x_{11} & 3x_{21}+\frac{5}{2}x_{20}x_{01} & 5x_{12} & -2x_{22} \\ x_{21} & 2x_{21} & \frac{5}{2}x_{31}-3x_{20}x_{11} & 2x_{22} & 5x_{12}x_{20} \\ x_{31} & 3x_{31} & \frac{1}{2}x_{20}x_{21} & 3x_{12}x_{20} & 0 \\ 2x_{12} & x_{12} & x_{22} & -2x_{12}x_{01} & 2x_{12}x_{11} \\ 2x_{22} & 2x_{22} & -2x_{31}x_{01}-x_{12}x_{20} & -3x_{12}x_{11} & 3x_{12}x_{21}+x_{11}x_{22} \\ & & -2x_{11}x_{21} & -3x_{01}x_{22} & \end{array}\right.$$

$$\left.\begin{array}{cc} -x_{31}+x_{20}x_{11} & 2x_{20}x_{21} \\ -2x_{12}-\frac{1}{2}x_{11}x_{01} & -2x_{22}-\frac{1}{2}x_{11}^2-2x_{01}x_{21} \\ -2x_{22}-\frac{1}{2}x_{01}x_{21} & \frac{3}{2}x_{31}x_{01}+7x_{12}x_{20} \end{array}\right) .$$

$$
\begin{array}{ll}
2x_{12}x_{20}+x_{11}x_{21}+\frac{1}{2}x_{31}x_{01} & 4x_{20}x_{22}+\frac{1}{2}x_{11}x_{31} \\
\frac{1}{2}x_{21}^{2} - 2x_{22}x_{20} & \frac{1}{2}x_{31}x_{21}+5x_{12}x_{20}^{2} \\
x_{12}x_{11} & 0 \\
x_{11}x_{22}+\frac{5}{2}x_{12}x_{21}+\frac{1}{2}x_{12}x_{20}x_{01} & \frac{5}{2}x_{12}x_{31}- \frac{11}{2}x_{12}x_{20}x_{11}
\end{array}
\right)
$$

$$
{}^{t}(X_{\cdot 0}\ ,\ X_{0\cdot}\ ,\ X_{10}\ ,\ X_{01}\ ,\ X_{11}^{(1)},\ X_{11}^{(2)},\ X_{21})={}^{t}L(x)\,{}^{t}(D_{20},\dots,D_{22}).
$$

	$X_{\cdot 0}$	$X_{0\cdot}$	X_{10}	X_{01}	$X_{11}^{(1)}$	$X_{11}^{(2)}$	X_{21}
$X(\log f)$	12	16	0	$-2x_{01}$	$2x_{11}$	$4x_{11}$	$-2x_{21}$
$\mathrm{div}^{(x)}X$	8	11	0	$-4x_{01}$	$2x_{11}$	$\frac{7}{2}x_{11}$	$\frac{1}{2}x_{21}$
$\mathrm{Tr}_{\mathfrak{g}}^{B}$ ad X	4	5	0	$0\ \ 2x_{01}$	0	$\frac{1}{2}x_{11}$	$-\frac{5}{2}x_{21}$

where $f = \det L(x)$. We omit the commutation relations.

$$
\sigma(f^{\alpha}) = \xi_{20}^{-8\alpha - \frac{11}{2}}\ \xi_{01}^{-12\alpha - 8}\ \sqrt{d\xi}\ /\ \sqrt{dx}\quad .
$$

Using $X_{\cdot 0}$, $X_{0\cdot}$, X_{10} and $\sigma(f^{\alpha})$, we get $b_{(7,7)}=1$.

Using $L(x)$, we see that rank $L(x) = 1$ if and only if $x=(c,0,\dots,0)$ or $(0,c,0,\dots,0)$, $c\neq 0$, which correspond to $S_{6,14,2}$ and $S_{6,8}$.

4.6. $S_{5,21}$

$v_0 = v_{12,10} + 346 + 148 + 247$

basis of $T^{*}_{v_0}V$: 156, 256+157, 356+158-257,357+258, 358

$$
p(x)= x_0<156>- \tfrac{1}{2}x_1<256>- x_2(<356>+<158>) + x_3(<357>+<258>)+ 4x_4<358>
$$

$$L(x) = \begin{pmatrix} 0 & 4x_0 & x_1 & 0 & 0 \\ x_1 & 3x_1 & 2x_2 & 4x_0 & \frac{1}{2}x_1x_2 - 3x_0\,x_3 \\ 2x_2 & 2x_2 & 3x_3 & 3x_1 & x_2^2 - 4x_0x_4 - 2x_1x_3 \\ 3x_3 & x_3 & 4x_4 & 2x_2 & \frac{1}{2}x_2x_3 - 3x_1x_4 \\ 4x_4 & 0 & 0 & x_3 & 0 \end{pmatrix},$$

$${}^t(X_0^{(1)},\ X_0^{(2)},\ X_{1-1},\ X_{-1\,1},\ X_{22}) = {}^tL(x)\,{}^t(D_0,\dots,D_4).$$

	$X_0^{(1)}$	$X_0^{(2)}$	X_{1-1}	$X_{-1\,1}$	X_{22}
$X(\log f)$	12	12	0	0	$4x_2$
$\mathrm{div}^{(x)}\ X$	10	10	0	0	$3x_2$
$\mathrm{Tr}_{\mathfrak{g}}^{B}$ ad X	2	2	0	0	x_2

$$[X_0^{(1)}, X_{ij}] = iX_{ij}\ ,\quad [X_0^{(2)}, X_{ij}] = jX_{ij}\ ,$$
$$[X_{1-1}\ ,\ X_{-11}] = X_0^{(1)} - X_0^{(2)}\ ,$$
$$[X_{1-1}\ ,\ X_{22}\ ] = \tfrac{1}{4}x_3X_0^{(1)} + \tfrac{3}{4}x_3X_0^{(2)} - \tfrac{1}{2}x_2X_{1-1} - x_4X_{-11}\ ,$$
$$[X_{-11}\ ,\ X_{22}\ ] = \tfrac{3}{4}x_3X_0^{(1)} + \tfrac{1}{4}x_3X_0^{(2)} - x_0X_{1-1} - \tfrac{1}{2}x_2X_{-11}\ .$$

$$b_{(5,21)} \mid (s+5/6)(s+7/6)\ .$$

4.7. $S_{5,12}$

$v_0 = v_{12,10} + 146 + 256 + 157$

basis of $T^*_{v_0}V$: 346+148-357-258, 247, 347+248, 348, 358

$p(x) = x_{20}\langle 247\rangle + x_{01}(\langle 346\rangle+\langle 148\rangle) + x_{11}(\langle 347\rangle+\langle 248\rangle) + x_{02}\langle 348\rangle$
$-x_{-12}\langle 358\rangle$.

$$L(x) = \begin{pmatrix} 2x_{20} & 0 & 0 & 0 & 2x_{11} \\ 0 & x_{01} & 3x_{11} & x_{02}-x_{01}^2 & x_{-12} \\ x_{11} & x_{11} & 3x_{20}x_{01} & x_{20}x_{-12} & x_{02} \\ 0 & 2x_{02} & 8x_{01}x_{11}+2x_{20}x_{-12} & 3x_{11}x_{-12} & 3x_{01}x_{-12} \\ -x_{-12} & 2x_{-12} & 2x_{02} & x_{01}x_{-12} & 0 \end{pmatrix},$$

$${}^t(X_0^{(1)},\ X_0^{(2)},\ X_{10}\ ,\ X_{01}\ ,\ X_{-11}) = {}^tL(x)\,{}^t(D_{20},\dots,D_{-12})\ .$$

$$X_0^{(1)} \quad X_0^{(2)} \quad X_{10} \quad X_{01} \quad X_{-11}$$

$X(\log f)$	2	8	0	$-2x_{01}$	0
$\mathrm{div}^{(x)}X$	2	6	0	$-x_{01}$	0
$Tr^B_{\mathscr{g}}$ ad X	0	2	0	$-x_{01}$	0

$$[X_0^{(1)}, X_{ij}] = iX_{ij} , \ [X_0^{(2)}, X_{ij}] = jX_{ij} ,$$
$$[X_{10}, X_{01}] = x_{11}(X_0^{(1)} - X_0^{(2)}) + x_{01}X_{10} - x_{20}X_{-11} ,$$
$$[X_{10}, X_{-11}] = 3x_{01}X_0^{(1)} - x_{01}X_0^{(2)} - X_{01} ,$$
$$[X_{01}, X_{-11}] = x_{-12}X_0^{(1)} .$$

$$\sigma(f^{\alpha}) = \xi_{20}^{-\alpha-1} \xi_{01}^{-8\alpha-6} \sqrt{d\xi} / \sqrt{dx} .$$

$$b_{(5,12)} = 1 .$$

4.7. $S_{6,14,0}$, $S_{6,14,2}$

$S_{6,14,0}$

linear part of the transverse localization

$$L'(x) = \begin{pmatrix} 2x_{222} & 2x_{222} & 2x_{222} & 0 & 0 & 0 \\ x_{131} & 3x_{131} & x_{131} & -2x_{222} & 0 & 0 \\ x_{113} & x_{113} & 3x_{113} & 0 & -2x_{222} & 0 \\ 2x_{200} & 0 & 0 & 0 & 0 & x_{222} \\ 0 & 4x_{040} & 0 & x_{131} & 0 & 0 \\ 0 & 0 & 4x_{004} & 0 & x_{113} & 0 \end{pmatrix} ,$$

$${}^t(X_0^{(1)}, X_0^{(2)}, X_0^{(3)}, X_{1-11}, X_{11-1}, X_{022}) = {}^tL'(x)\, {}^t(D_{222}, \ldots, D_{004}) .$$

	$X_0^{(1)}$	$X_0^{(2)}$	$X_0^{(3)}$
$X(\log f)$	8	12	12
$\mathrm{div}^{(x)}X$	6	10	10
$Tr^B_{\mathscr{g}}$ ad X	2	2	2

$$\sigma(f^{\alpha}) = \xi_{200}^{-4\alpha-3} (\xi_{040}\xi_{004})^{-3\alpha-\frac{5}{2}} \sqrt{d\xi} / \sqrt{dx} .$$

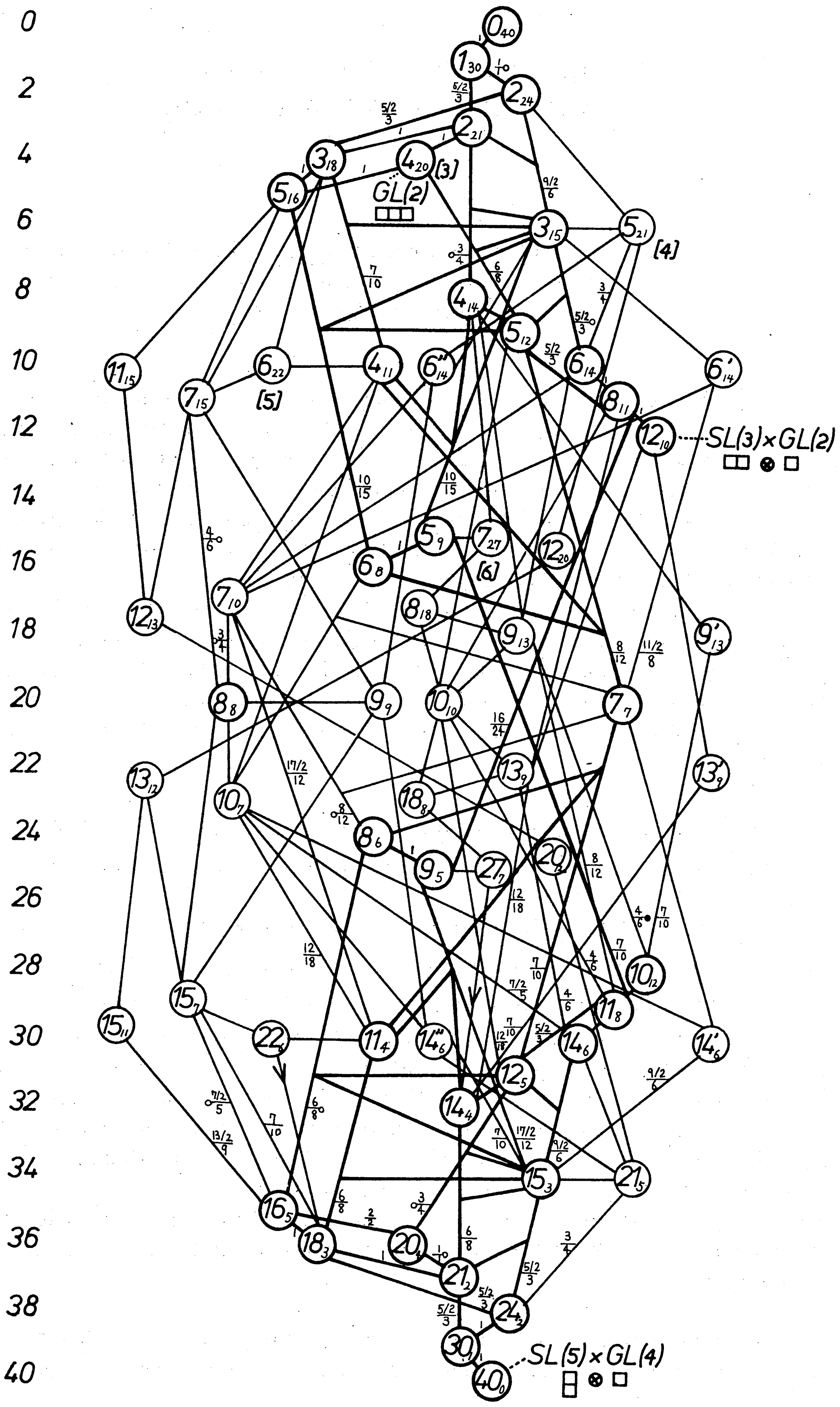
$S_{6,14,2}$

Linear part of the transverse localization

$$L'(x) = \begin{pmatrix} 2x_{20} & 0 & 0 & 0 & 0 & 0 \\ 0 & 2x_{02} & 0 & 0 & 0 & 0 \\ 3x_{3-2} & -2x_{3-2} & x_{20} & 0 & 0 & 0 \\ -2x_{-23} & 3x_{-23} & 0 & x_{02} & 0 & 0 \\ 0 & x_{01} & 0 & -3x_{20} & 0 & x_{02} \\ x_{10} & 0 & 3x_{02} & 0 & x_{20} & 0 \end{pmatrix},$$

$$^t(X_0^{(1)}, X_0^{(2)}, X_{-12}, X_{2-1}, X_{10}, X_{01}) = {}^tL'(x)\,{}^t(D_{20},\dots,D_{10}) .$$

	$X_0^{(1)}$	$X_0^{(2)}$
$X(\log f)$	6	6
$\mathrm{div}^{(x)}X$	4	4
$\mathrm{Tr}_{\mathfrak{g}}^{B}\ \mathrm{ad}\ X$	2	2


GL(2)
SL(3)×GL(2)
SL(5)×GL(4)
[3]
[4]
[5]
[6]